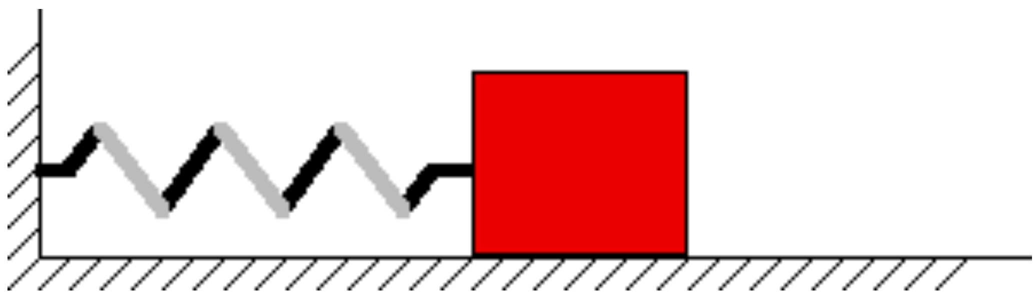
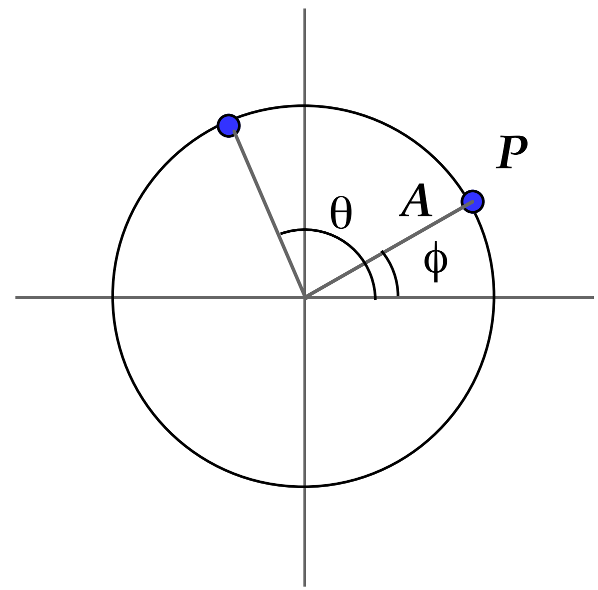
Ch 15
Simple Harmonic Motion



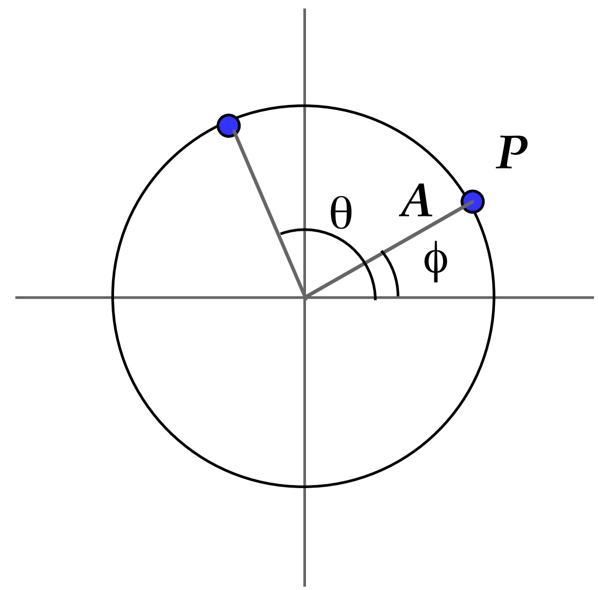
Periodic (Circular) Motion

Point *P* is travelling in a circle with a constant speed. How can we determine the *x*-coordinate of the point *P* in terms of other given quantities?



Periodic (Circular) Motion

- Point P is beginning its circular motion at an arbitrary angular position ϕ.
- As P rotates with some angular velocity w, the angular position of P is given by $\theta = \omega t + \phi$.
- What is the *x* coordinate of *P* as it rotates around the circle?

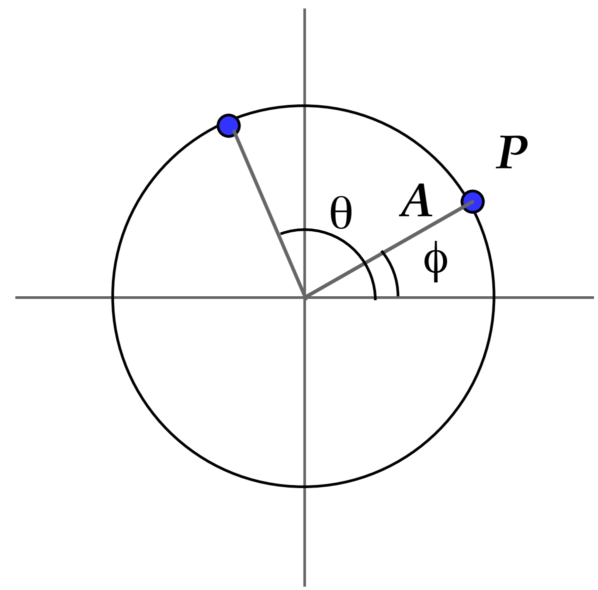


$$x = A\cos\theta$$

$$x = A\cos(\omega t + \phi)$$

Vocabulary

A = the "amplitude" of the motion



T = the "period" = the time for one complete revolution

ω = the angular speed of P = "angular frequency"

ϕ = the "phase constant"

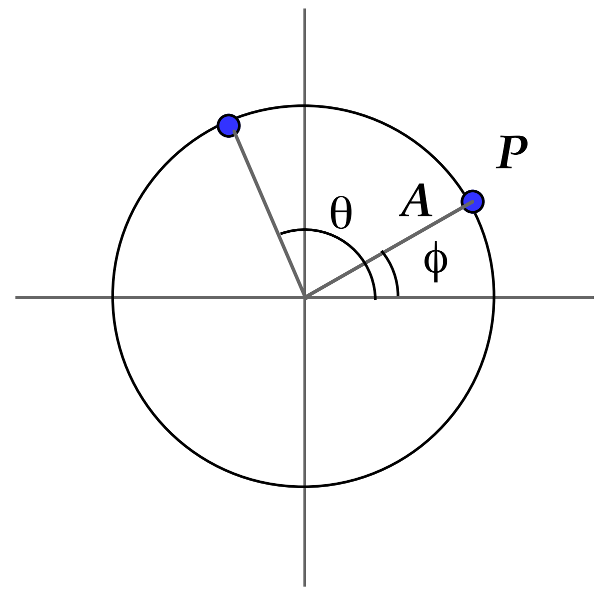
$$x = A\cos\theta$$

$$x = A\cos(\omega t + \phi)$$

S.H.M.

By definition, a particle on the x-axis exhibits simple harmonic motion when its position varies according to this relationship:

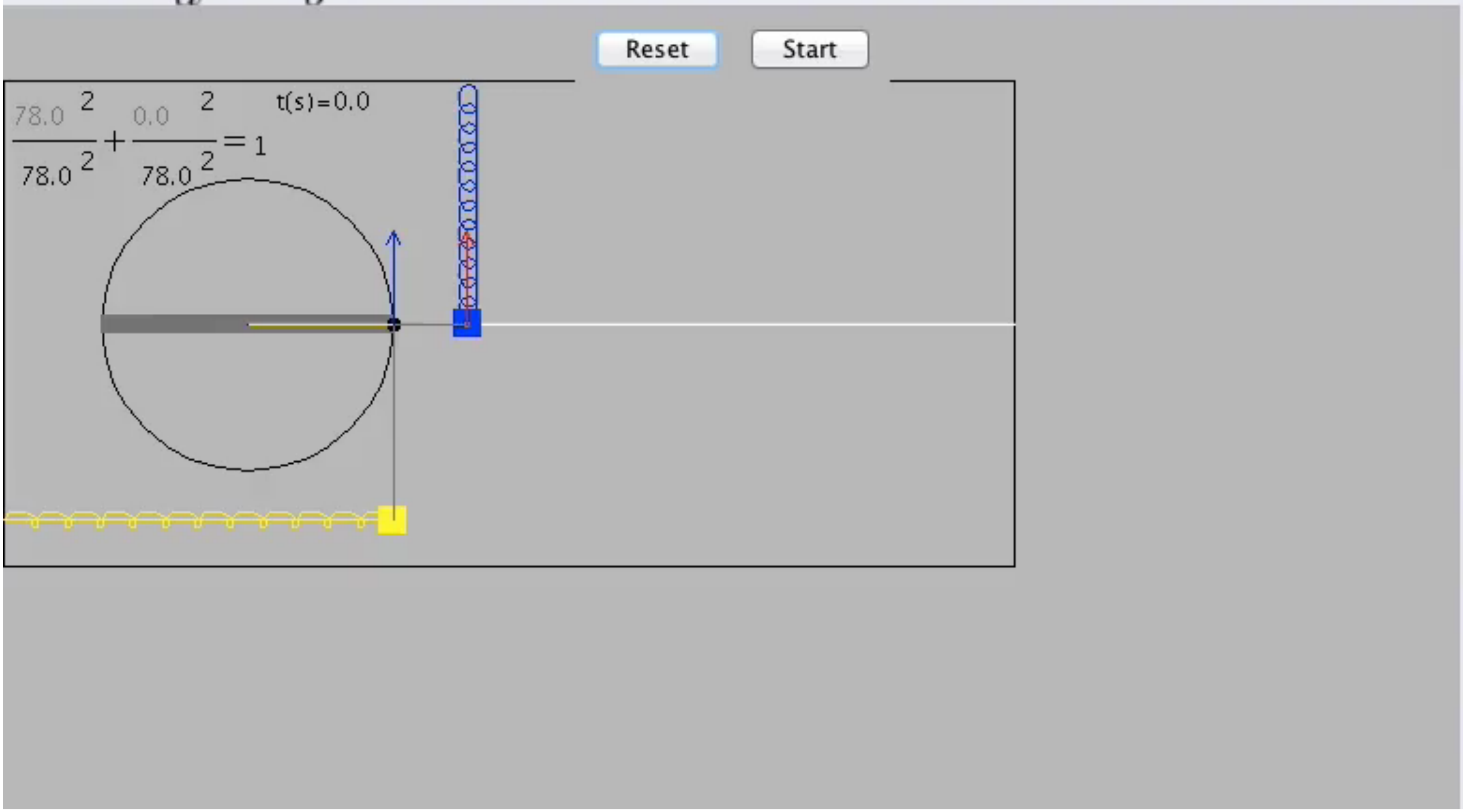
$$x = A\cos(\omega t + \phi)$$



x is a function of *t*, and repeats when ωt increases by 2π.

S.H.M.

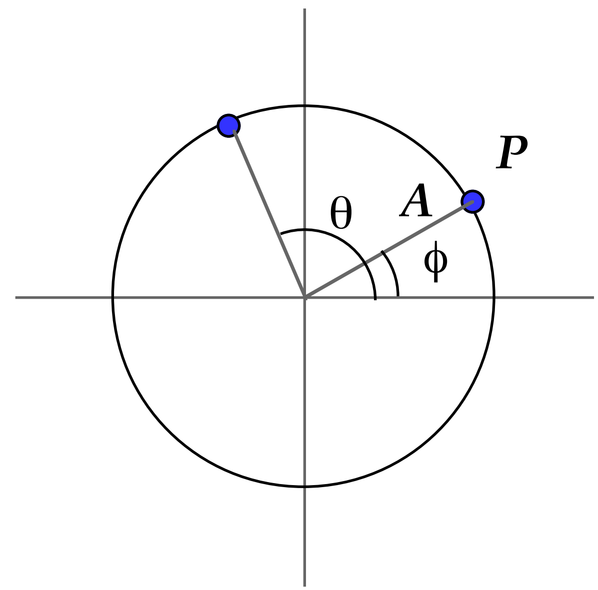
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, x(t) = a * \sin(\omega t), y(t) = b * \cos(\omega t)$$



Frequency

What is the relationship between *T* and ω?

$$T = \frac{2\pi}{\omega}$$



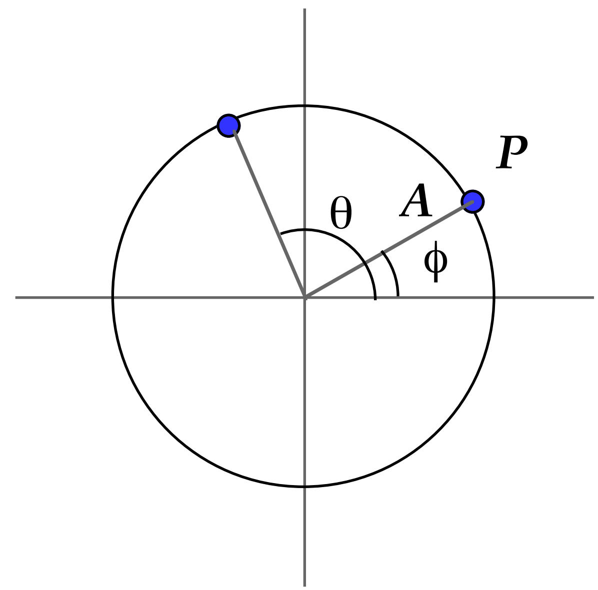
$$x = A\cos(\omega t + \phi)$$

The units for *T* are obviously "seconds" (per one cycle). If we invert *T*, we get *frequency* ***f***, which is in "cycles per second," or "Hertz."

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

Derive v(t)?

$$x = A\cos(\omega t + \phi)$$



$$v = \frac{dx}{dt}$$

$$v = \frac{d}{dt}(A\cos(\omega t + \phi))$$

$$v = -\omega A\sin(\omega t + \phi)$$

And what is *v*'s maximum value?

$$v_{\text{max}} = -\omega A$$

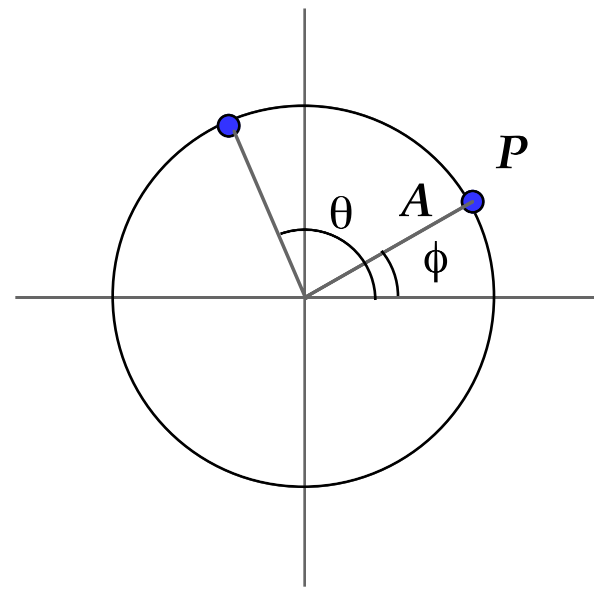
Derive *a*(t)?

$$a = -\omega A\sin(\omega t + \phi)$$

$$a = \frac{dv}{dt}$$

$$a = \frac{d}{dt}(-\omega A\sin(\omega t + \phi))$$

$$a = -\omega^2 A\cos(\omega t + \phi)$$



Note this important relationship:

$$a = -\omega^2 x$$

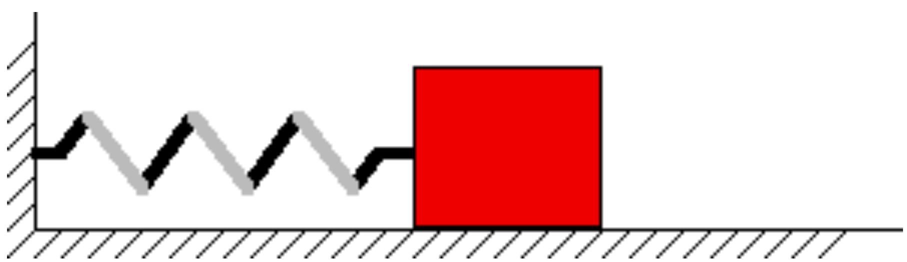
Example 1

Does the acceleration of a simple harmonic oscillator remain constant during its motion?

Is the acceleration of a simple harmonic oscillator ever zero?

Mass-Spring = SHM?

Do mass-spring systems exhibit SHM? How would we know?



1. Assuming it's a Hooke's Law spring, we know that F_{spring}=-kx, or ma=-kx, or a=-(k/m)x, where k/m is a constant value.

2. We've already established, for SHM, that a=-ω^2Acos(ωt+ϕ), so

   a=-ω_2x

3. Therefore, if -ω^2x= -(k/m)x, our mass-spring system exhibits SHM.

f & *T* for Mass-Spring

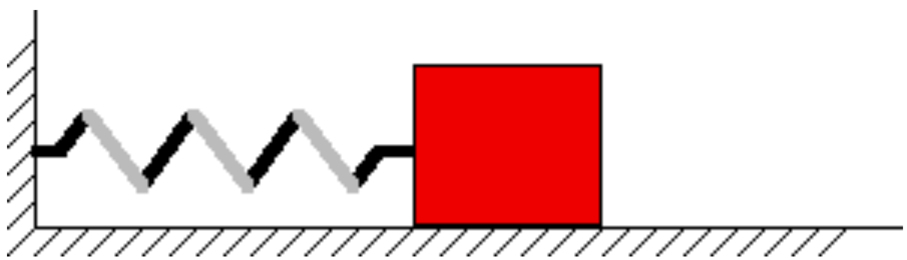
$$T = \frac{2\pi}{\omega}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$T = \frac{2\pi}{\sqrt{\frac{k}{m}}} = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Mass-Spring Energy



$$E = U_{\max} = \frac{1}{2}kA^2$$

$$E = K_{\max} = \frac{1}{2}m{v_o}^2$$

$$E = \frac{1}{2}m{v_x}^2 + \frac{1}{2}kx^2$$

Example 2

A 5.00-kg mass is connected to a light spring with k=20 N/m, and oscillating on a horizontal frictionless surface.

a. Calculate total energy of the system and speed of mass at x_0 if Amplitude is 3.00 cm.

b. What is the velocity of the mass at x=2.00 cm?

c. Find K & U when x = 2.00 cm

d. For what values of x does the speed of the mass = 0.1 m/s?

The Simple Pendulum

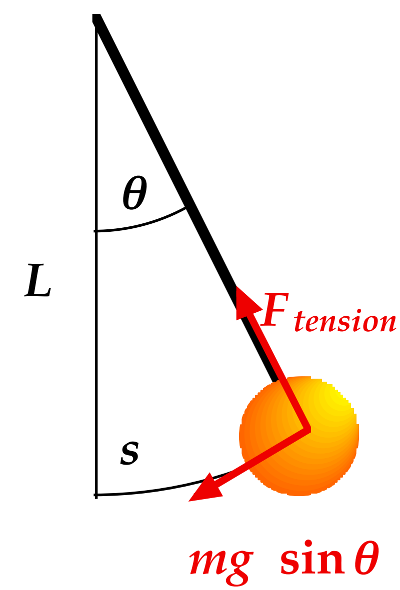
... consists of a mass (a "bob") at the end of a string of length *L*, swinging back and forth due to the effect of gravity.

$$F_{\text{tangential}} = -mg\sin\theta = m\frac{d^2s}{dt^2}$$

$$s = L\theta, so$$

$$-g\sin\theta = \frac{d^2L\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} = \frac{-g}{L}\sin\theta$$



Pendulum = SHM?

1. Set calculator mode to rads
2. Set graph range for 0-1, 0-1
3. Graph y1=x
4. Graph y2=sin x

The Physical Pendulum

A physical pendulum includes a solid object of mass *m* that oscillates under the influence of gravity.

$$\tau = -mgd\sin\theta \quad \text{and} \quad \tau = I\alpha$$

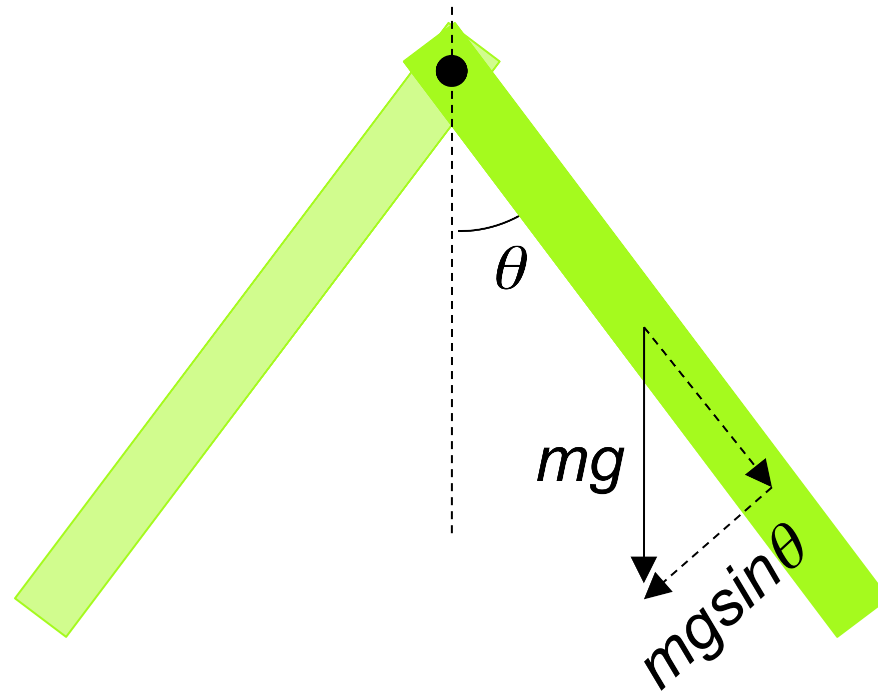
$$-mgd\sin\theta = I\alpha$$

$$-mgd\sin\theta = I\frac{d^2\theta}{dt^2}$$

Assuming small θ :

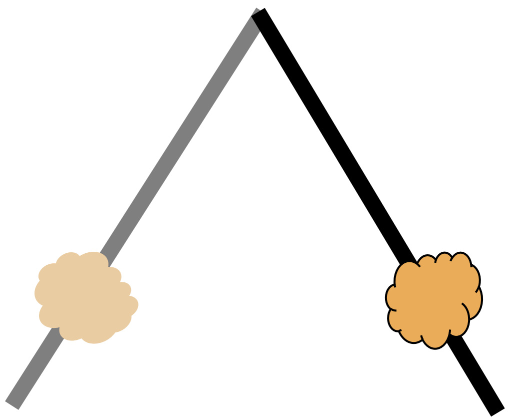
$$\frac{d^2\theta}{dt^2} = \frac{-mgd\theta}{I}, \quad so \quad \omega = \sqrt{\frac{mgd}{I}}$$

(where d = distance from rotation axis to cg)



Example 3

A pendulum is made of a 1-kg meter stick with a 2-kg blob of clay stuck on it at the 75-cm mark.



a. What is the moment of inertia of this pendulum?

$$I_{total} = I_{stick} + I_{bob} = \frac{1}{3}ML^2 + MR^2$$

$$I = \frac{1}{3}(1kg)(1m)^2 + (2kg)(0.75m)^2$$

$$I = 1.46 kg \bullet m^2$$

b. Where is the pendulum arm's center of mass located?

$$x_{cm} = \frac{1}{M}\left(x_1 m_1 + x_2 m_2\right)$$

$$x_{cm} = \frac{1}{3kg}\left((0.5m)(1kg) + (0.75m)(2kg)\right)$$

$$x_{cm} = 0.67m$$

c. What is the period of the pendulum as it swings from an axis at the top end?

$$\omega = \sqrt{\frac{mgd}{I}} \text{ (from previous page)}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgd}}\ldots$$

Ch 10-11 Test

Test average 2015-2016: 67.5%

Test average 2014-2015: 77.3%

Test average 2013-2014: 71.1%

Test average 2012-2013: 77.6%

HW Stats for 2012-2013:

Avg for stdnts regularly completing homework: 83.3%

Avg for stdnts with late or missing assignments: 71.6%

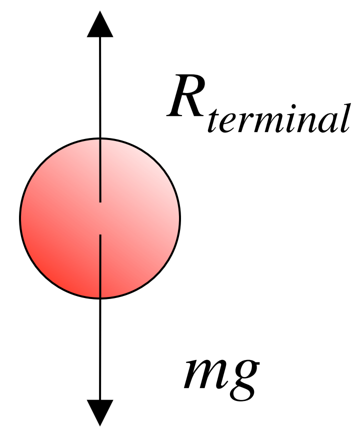
Test average 2011-2012: 79.9%

Test average 2010-2011: 69.7%

Finklebottom & Priest

"In the presence of air friction, objects accelerate at a constant rate $k < g$."

If this is the case, what will the graph of *x* vs. *t* look like for an object falling in air? Is this what we actually observe?



$$R = -bv \qquad \text{or} \qquad R = \frac{1}{2} D\rho v^2$$

Two common models of air friction. Which behavior is the cotton exhibiting? How could you find out?

$$R \propto v \qquad \text{or} \qquad R = \propto v^2$$

Graphing exercise

1. Set calculator mode to radians, and Standard Zoom on graph window
2. Graph $x=A\cos(\omega t+\phi)$ as y1=4 cos (1x+0)
3. Graph $v=-\omega A\sin(\omega t+\phi)$ as y2=-1•4•sin(1*x*+0)
4. Graph *a* as well.

NOTES

1. *x*, *v*, and *a* all vary sinusoidally with time, but are not "in phase"

2. *a* is proportional to *x*, but in the opposite direction

3. *f* and *T* are independent of *A*