A NOTE TO THE STUDENTS

Drift velocity? This is a little obscure. Follow along.

The assumption I have made in generating these solutions is that you have spent time trying to fight your way through a problem and have gotten yourself balled up in the process. Although this won't be the case with the easier problems, the commentary I'm providing with these solutions is designed to help people in that situation understand where and why they went sideways.

If it is the case that you haven't spent much time and/or effort on a given problem, or if you are using the problems to reverse engineer the physics (that is, your are not internalizing the physics, then using that understanding to unravel the problems but, rather, are using the problems in an attempt *to* learn the physics), the commentary will in all probability not be as useful to you as might otherwise have been. In fact, its presence may even be a bit irritating. If that be the case, I apologize. The intent was not to make your life harder than it already is.

In any case, and this is especially true if you've worked a problem out on your own to conclusions, you are always free to skip any commentary that is provided and simply jump to the bottom line.

Problem 27.3

Drift velocity? This is a little obscure. Follow along.

The equation that relates the drift velocity to the current "I" is:

$$I = nqAv_d$$

where *n* be the number of charge carriers per unit volume (and in this case, that number is also the number of free charges in this case), *q* the charge on one charge and *A* the cross-sectional area of the wire.

Knowing the mass density ρ (this is in grams/cm^3) of aluminum, we can write:

$$\rho = nm$$

where *n is the number of atoms per volume* and *m* is the mass of an atom. (Obscure? Yes, but that's why you are looking at the solutions.). In other words, given ρ , we can determine *n* if we can figure out the mass of each atom. That we can do by using Avogadro's number and the fact that the molar mass of aluminum is 27 grams. (Again, obscure? You bet!) Using all of that information, we can write:

The mass per atom:

$$m = \frac{27 \text{ grams}}{6.02 x 10^{23}}$$
$$= 4.49 x 10^{-23}$$

With the mass per atom, we can write:

$$\rho = nm$$
$$\Rightarrow \quad n = \frac{\rho}{m}$$
$$\Rightarrow \quad n = \frac{2.7 \text{ g/cm}^3}{4.49 x 10^{-23} \text{ g/atom}}$$
$$= 6.02 x 10^{22} \text{ atoms/cm}^3$$
$$= 6.02 x 10^{28} \text{ atoms/m}^3$$

With all that, we can write:

$$I = nqAv_d$$

$$\Rightarrow \quad v_d = \frac{I}{nqA}$$

$$\Rightarrow \quad v_d = \frac{5A}{\left(6.02x10^{28}\,m^{-3}\right)\left(1.6x10^{-19}\,C/\right)\left(4x10^{-6}\,m^{2}\right)}$$

$$= .13 \text{ m/s}$$

I doubt you will ever see a problem like this on an AP test. Its use, as best I can tell, is really only to make you think outside the box . . . (which is never bad).

Problem 27.5

Assume $I(t) = I_o e^{-t/\tau}$

a.) How much charge passes between t = 0 and τ ?

$$I(t) = \frac{dq(t)}{dt}$$

$$\Rightarrow \quad Q = \int dq = \int I\,dt$$

$$\Rightarrow \quad Q = \int_{t=0}^{\tau} \left(I_o e^{-t/\tau}\right) dt$$

$$\Rightarrow \quad Q = -I_o \tau e^{-t/\tau}\Big|_{t=0}^{\tau}$$

$$\Rightarrow \quad Q = -I_o \tau \left(e^{-\tau/\tau} - e^{-0/\tau}\right)$$

$$\Rightarrow \quad Q = I_o \tau \left(1 - \frac{1}{e}\right)$$

$$\Rightarrow \quad Q = .632 I_o \tau$$

Problem 27.5

Assume $I(t) = I_o e^{-t/\tau}$

b.) How much charge passes between t = 0 and 10τ ?

$$I(t) = \frac{dq(t)}{dt}$$

$$\Rightarrow \quad Q = \int dq = \int I\,dt$$

$$\Rightarrow \quad Q = \int_{t=0}^{10\tau} \left(I_o e^{-t/\tau}\right) dt$$

$$\Rightarrow \quad Q = -I_o \tau e^{-t/\tau} \Big|_{t=0}^{10\tau}$$

$$\Rightarrow \quad Q = -I_o \tau \left(e^{-10\tau/\tau} - e^{-0/\tau}\right)$$

$$\Rightarrow \quad Q = I_o \tau \left(1 - \frac{1}{e^{10}}\right)$$

$$\Rightarrow \quad Q = .99995 I_o \tau$$

Problem 27.5

Assume $I(t) = I_o e^{-t/\tau}$

c.) How much charge passes between t = 0 and 10τ?

$$I(t) = \frac{dq(t)}{dt}$$

$$\Rightarrow \quad Q = \int dq = \int I\,dt$$

$$\Rightarrow \quad Q = \int_{t=0}^{\infty} \left(I_o e^{-t/\tau}\right) dt$$

$$\Rightarrow \quad Q = -I_o \tau e^{-t/\tau} \Big|_{t=0}^{\infty}$$

$$\Rightarrow \quad Q = -I_o \tau \left(e^{-\infty/\tau} - e^{-0/\tau}\right)$$

$$\Rightarrow \quad Q = I_o \tau \left(1 - \frac{1}{e^{\infty}}\right)$$

$$\Rightarrow \quad Q = I_o \tau$$

Problem 27.11

a.) The current density (current per unit area) is:

$$J = \frac{I}{A} = \frac{8x10^{-6}\,A}{\pi\left(10^{-3}\,m\right)^2}$$

$$= 2.55\ A/m^2$$

b.) From Problem 27.3, we can write the electron density *n* as:

$$I = nqAv_d$$

$$\Rightarrow \quad J = \frac{I}{A} = nev_d$$

$$\Rightarrow \quad n = \frac{J}{ev_d}$$

$$\Rightarrow \quad = \frac{\left(2.55\ A/m^2\right)}{\left(1.6x10^{-19}C\right)\left(3x10^{8}\ m/s\right)}$$

$$\Rightarrow \quad = 5.3x10^{10}\ m^{-3}$$

c.) How long will Avogadro's number of electrons exit the accelerator?

$$I = \frac{\Delta q}{\Delta t}$$

$$\Rightarrow \quad \Delta t = \frac{\Delta q}{I}$$

$$= \frac{\left(N_{Avogadro} e\right)}{I}$$

$$= \frac{\left(6.02x10^{23}\right)\left(1.6x10^{-19} C\right)}{\left(8x10^{-6} A\right)}$$

$$= 1.2x10^{10} \text{ seconds}$$

Problem 27.12

The voltage difference across a resistor is proportional to the current through the resistor and the resistance of the resistor, or:

$$V_R = iR$$

$$\Rightarrow \quad i = \frac{V_R}{R}$$

$$= \frac{(120\ V)}{(240\ \Omega)}$$

$$= .5\ \text{amps}$$

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Problem 27.14

The resistance of a wire is proportional to the length of the wire (the longer the wire, the more the resistance), inversely proportional to the cross-sectional area of the wire (the smaller the area, the bigger the resistance to current flow) with a proportionality constant equal to the material's resistivity. The resistivity is found in a table in the book. Doing the math, we get:

$$
\begin{aligned}
R &= \rho \frac{L}{A} \\
&= \left(5.6\text{x}10^{-8}\,\Omega \bullet \text{m}\right)\frac{\left(1.5\ \text{m}\right)}{\left(.6\text{x}10^{-6}\,\text{m}^2\right)} \\
&= 1.4\text{x}10^{-1}\ \Omega
\end{aligned}
$$

The current is:

$$
\begin{aligned}
i &= \frac{V_R}{R} \\
&= \frac{\left(.9\ \text{V}\right)}{\left(1.4\text{x}10^{-1}\ \Omega\right)} \\
&= 6.43\ \text{amps}
\end{aligned}
$$

Problem 27.15

You are given the copper wire's mass and resistance. This one of those "take the relationships we know are true and manipulate the daylights out of them!" problems. To that end, then, to determine length and cross-sectional area.

If we take the mass density to be ρ_m in kg/cubit-meters, we can write:

$$m = \left(\rho_m \ \text{kg/m}^3\right)\left(V \ \text{m}^3\right)$$
$$= \left(\rho_m \ \text{kg/m}^3\right)\left[\left(A \ \text{m}^2\right)\left(L \ \text{m}\right)\right]$$
$$\Rightarrow \ A = \frac{m}{\rho_m L}$$

Taking resistivity to be ρ (yes, an inconvenient symbol, but that's its symbol), we can write the resistance as:

$$R = \rho \frac{L}{A} = \rho \frac{L}{\left(m/\rho_m L\right)}$$

$$= \rho \frac{L}{\left(m/\rho_m L\right)} = \frac{\rho \rho_m L^2}{m}$$

a.) the length?

Continuing:

$$R = \frac{\rho\rho_m L^2}{m}$$

$$\Rightarrow \quad L = \left(\frac{Rm}{\rho\rho_m}\right)^{1/2}$$

$$\Rightarrow \quad L = \left(\frac{(.5\text{meters})(10^{-3}\text{kg})}{(1.7\text{x}10^{-8})(8.92\text{x}10^{3})}\right)^{1/2}$$

$$= 1.82 \text{ m}$$

b.) The wire radius? Going back to the relationship between the mass, mass density and volume, we can write:

$$m = \rho_m V \quad \Rightarrow \quad V = \frac{m}{\rho_m}$$

$$\Rightarrow \quad (\pi r^2 L) = \frac{m}{\rho_m}$$

Continuing:

$$\left(\pi r^2 L\right) = \frac{m}{\rho_m}$$

$$\Rightarrow \quad r = \left(\frac{m}{\pi \rho_m L}\right)^{1/2}$$

$$= \left(\frac{\left(10^{-3}\right)}{\pi\left(8.92\text{x}10^{3}\right)\left(1.82\right)}\right)^{1/2}$$

$$= 1.4\text{x}10^{-4} \text{ m with a diameter twice that value}$$

Problem 27.23

The temperature dependence of a resistor is defined as:

$$R = R_o\left[1 + \alpha\left(T - T_o\right)\right]$$

$$\Rightarrow \quad R - R_o = \alpha R_o\left(T - T_o\right)$$

$$\Rightarrow \quad \frac{R - R_o}{R_o} = \alpha\Delta T$$

$$= \left(5x10^{-3}\right)\left(25\right)$$

$$= .12$$

Problem 27.27

According to the text, the temperature dependence of resistivity, as applied to aluminum, is

$$\rho_{Al} = (\rho_o)_{Al}\left[1+\alpha_{Al}(T-T_o)\right]$$

where ρ_o is the resistivity at temperature T_o (room temperature) and α_{Al} is aluminum's *temperature coefficient of resistance*.

If $\rho_{Al} = 3(\rho_o)_{Cu}$, we can write:

$$\rho_{Al} = (\rho_o)_{Al}\left[1+\alpha_{Al}(T-T_o)\right] = 3(\rho_o)_{Cu}$$

$$\Rightarrow \quad T - T_o = \frac{1}{\alpha_{Al}}\left[\frac{3(\rho_o)_{Cu}}{(\rho_o)_{Al}} - 1\right]$$

$$\Rightarrow \quad T - (20^\circ C) = \frac{1}{\left(3.9\text{x}10^{-3}\,({}^\circ C)^{-1}\right)}\left[\frac{3(1.7\text{x}10^{-8}\ \Omega\bullet m)}{(2.82\text{x}10^{-8}\ \Omega\bullet m)} - 1\right]$$

$$\Rightarrow \quad T = 227^\circ C$$

Again, obscure!